The return-maximizing blend of reusable items/services with stochastic demand and providers with price differentials

Saga to "Optimizing delivery costs by establishing a new fleet size under stochastic demand" and "Minimizing cost of calls to wireless phones under Calling Party Pays"

E. González L. Epstein M. Herrera

[1] Facultad de Ingeniería y Ciencias
Universidad Adolfo Ibáñez
Santiago de Chile

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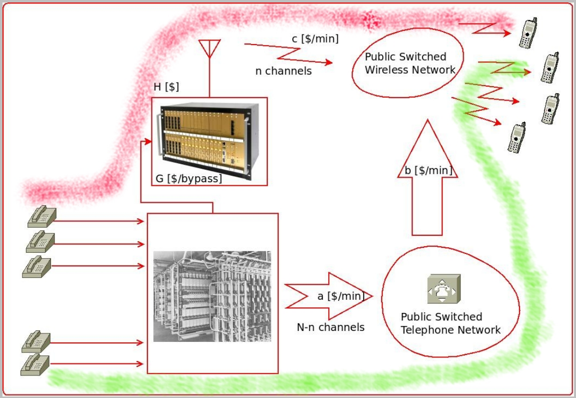
Context

Regulatory framework

- Legally binding: form Receiving Calling Party Pays to Calling Party Pays
- Differential of prices: High fixed cost & Low Op. Cost vs. Low fixed cost & igh Op. Cost
- Civil suits due to bypassing

Available bypass technology



Main Goal

- Reduce costs without loosing outbound calls.
- Include transient effects.

Method of solution

Already done

- Infinite population and stationary.
- Finite population and stationary.

Finite population and Transient

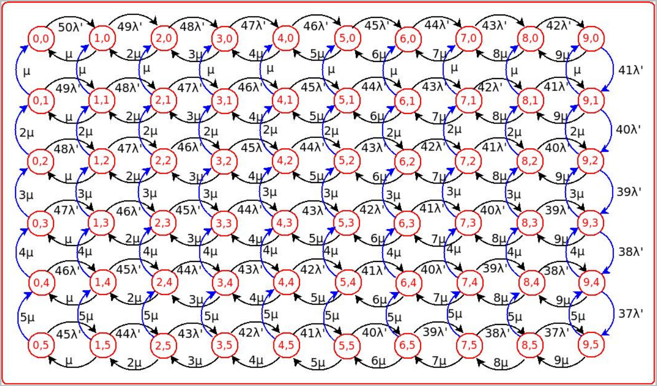
Analogies and assumptions

- Already existing (old trucks) with high operation costs. (Fixed to mobile costs)
- New fleet of trucks to buy: low operation costs, but non negligible purchase cost. (Mobile to Mobile costs)
- Both operationally equivalent trucks (new and old). (Same QoS as in Mobile traffic)
- Finite number of drivers follow an availability hourly-profile. (Like calling agents from a finite population)

Modelling calls to mobile (1)

Only to-mobile-calls and finite number of calling agents

Full states transition diagram for 9 bypasses, 5 overflow channels, 50 calling agents and negligible traffic to fixed phones.



Non stationary case

In general, from the transition diagram:

$$p_{i,j}(t+\Delta t)-p_{i,j}(t) = \left((i+1)\cdot\mu\cdot p_{i+1,j}+(j+1)\cdot\mu\cdot p_{i,j+1}+A_{i,j}\cdot p_{i-1,j} - p_{i,j}\cdot\left(j\cdot\mu+B_{i,j}+i\cdot\mu\right)\right)\Delta t$$

where

$$A_{i,j}=\{K-(i+j-1)\}\cdot\alpha$$

$$B_{i,j}=\{K-(i+j)\}\cdot\alpha$$

(and $K=50$).

Non stationary case

And the matrix equation resulting from all the possible states is:

$$\frac{dp}{dt} = Qp$$

where p stores all the transient states probabilities, and Q is a matrix that holds the coefficients of the linear equations already shown. Q can be any time function in which case the analytic solution is:

$$p(t) = e^{\int_0^t Q(\tau)d\tau} p(0)$$

And if time is discretized $Q(t)$ can be assumed as constant values changing at each time-step.

Mathematical Modelling

With m overflow channels, n bypass channels, K_h calling agents just to-mobile calls, the expected cost is:

$$C(n,m) = D \cdot L \cdot \left\{ \sum_{h=1}^{h=1440} \sum_{i=0}^{i=n} \sum_{j=0}^{j=m} (i \cdot V_{1h} + j \cdot V_{2h}) \cdot p(i,j,E_h,n,m,k_h) \right\} + n \cdot C_n + m \cdot C_m + CI \,,$$

where

$$L = \sum_{l=1}^{l=T} \left(\frac{1-b}{1+r} \right)^{l-1}$$

The expected traffic for one day of operation is,

$$T(n,m) = \frac{1}{60} \sum_{h=1}^{h=1440} \sum_{i=0}^{i=n} \sum_{j=0}^{j=m} (i+j) \cdot p_m(i,j,E_h,n,m,k_h) \cdot$$

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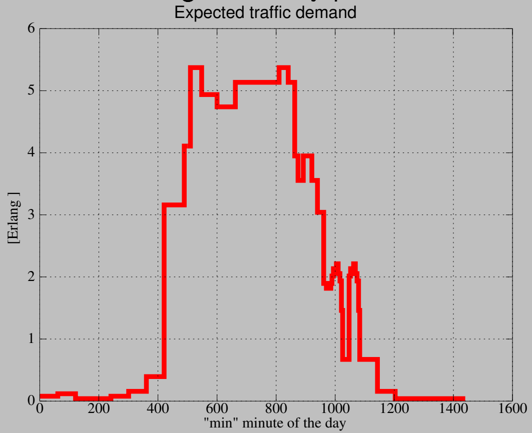
Input parameters (1)

With constant service rate and constant arrival call rate, but varying size of the population for one day, the expected demanded traffic assuming stationary probabilities is :



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Input parameters (3)

Main components

- Horizon (T): 24 months, (Extend)
- Working Days per month (D): 22 (Adjust)
- Actualization rate (r): 0.4% per month (Adjust)
- Tariff Reduction rate (b): 0.3% per month (Adjust)
- Finite and hourly-dependent number of calling agents (Adjust)
- Rates: (Adjust)
  - Day, land to mobile: 153.0 CLP/min
  - Night, land to mobile: 92.5 CLP/min
  - All day, mobile to mobile: 47.2 CLP/min
- C_{MFB}: $240 \cdot 10^3$CLP (Adjust)

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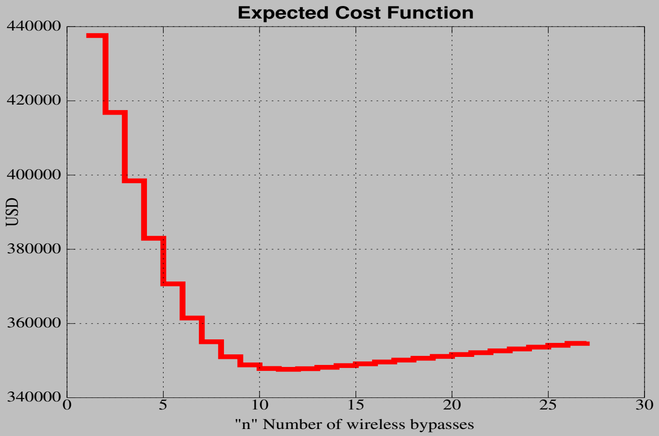
Optimization Example

Simplified model: Poisson in-frame homogeneous



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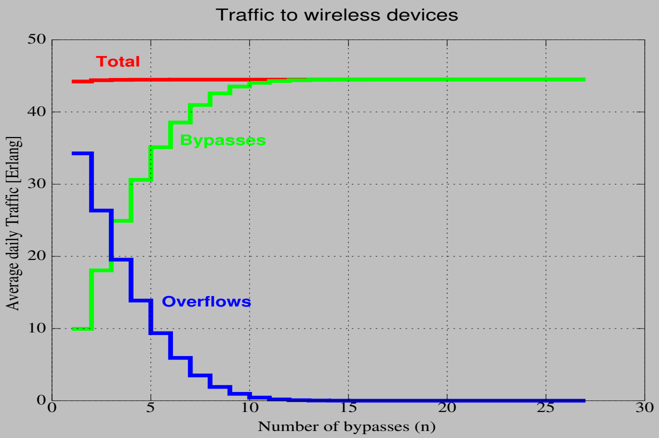
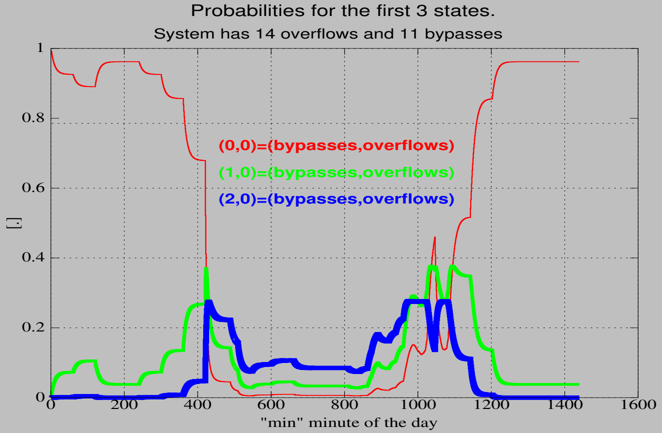


Probabilites



Existing 14 Overflow lines

- Analytical model:
  - Optimum numer of bypass: 11
    - Savings: 0.12 million USD
    - Optimum Cost: 0.348 million USD
    - Fast ROI

Traffic Profiles

Transient analysis shows the difference (green is transient):

Research follow-up

- Adjust values to other industries
- Three state variable model
- Design for Multicompany outsourcer
- Find criterion for simplifications
- Extend to other distributions of probability