The return-maximizing blend of reusable items/services with stochastic demand and providers with price differentials

Saga to “Optimizing delivery costs by establishing a new fleet size under stochastic demand” and “Minimizing cost of calls to wireless phones under Calling Party Pays”

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SAT-6, 2012
Context

Regulatory framework

- Legally binding: form Receiving Calling Party Pays to Calling Party Pays
- Civil suits due to bypassing
Available bypass technology
Main Goal

- Reduce costs without loosing outbound calls.
- Include transient effects.
Method of solution

Already done
- Infinite population and stationary.
- Finite population and stationary.

Finite population and Transient
Already existing (old trucks) with high operation costs. (Fixed to mobile costs)
New fleet of trucks to buy: low operation costs, but non negligible purchase cost. (Mobile to Mobile costs)
Both operationally equivalent trucks (new and old). (Same QoS as in Mobile traffic)
Finite number of drivers follow an availability hourly-profile. (Like calling agents from a finite population)
Modelling calls to mobile (1)
Only to-mobile-calls and finite number of calling agents

Full states transition diagram for 9 bypasses, 5 overflow channels, 50 calling agents and negligible traffic to fixed phones.
Non stationary case

In general, from the transition diagram:

\[ p_{i,j}(t + \Delta t) - p_{i,j}(t) = \left( (i+1) \cdot \mu \cdot p_{i+1,j} + (j+1) \cdot \mu \cdot p_{i,j+1} + A_{i,j} \cdot p_{i-1,j} \\
- p_{i,j} \cdot (j \cdot \mu + B_{i,j} + i \cdot \mu) \right) \Delta t \]

where

\[ A_{i,j} = \{ K - (i+j-1) \} \cdot \alpha \]

\[ B_{i,j} = \{ K - (i+j) \} \cdot \alpha \]

(and \( K = 50 \)).
Non stationary case

And the matrix equation resulting from all the possible states is:

\[
\frac{dp}{dt} = Qp
\]

where \( p \) stores all the transient states probabilities, and \( Q \) is a matrix that holds the coefficients of the linear equations already shown. \( Q \) can be any time function in which case the analytic solution is:

\[
p(t) = e^{\int_0^t Q(\tau)d\tau} p(0)
\]

And if time is discretized \( Q(t) \) can be assumed as constant values changing at each time-step.
With \( m \) overflow channels, \( n \) bypass channels, \( K_h \) calling agents just to-mobile calls, the expected cost is:

\[
C(n, m) = D \cdot L \cdot \left\{ \sum_{h=1}^{1440} \sum_{i=0}^{n} \sum_{j=0}^{m} \left( i \cdot V_{1h} + j \cdot V_{2h} \right) \cdot p(i, j, E_h, n, m, k_h) \right\} \\
+ n \cdot C_n + m \cdot C_m + C_l,
\]

where

\[
L = \sum_{l=1}^{T} \frac{\left( 1 - b \right)^{l-1}}{1 + r}
\]

The expected traffic for one day of operation is,

\[
T(n, m) = \frac{1}{60} \sum_{h=1}^{1440} \sum_{i=0}^{n} \sum_{j=0}^{m} (i + j) \cdot p_m(i, j, E_h, n, m, k_h).
\]
With constant service rate and constant arrival call rate, but varying size of the population for one day, the expected demanded traffic assuming stationary probabilities is:

$$\text{Expected traffic demand}$$

![Graph showing expected traffic demand over time.](image-url)
Input parameters (3)

Main components

- Horizon (T): 24 months, (Extend)
- Working Days per month (D): 22 (Adjust)
- Actualization rate (r): 0.4% per month (Adjust)
- Tariff Reduction rate (b): 0.3% per month (Adjust)
- Finite and hourly-dependent number of calling agents (Adjust)
- Rates: (Adjust)
  - Day, land to mobile: 153.0 CLP/min
  - Night, land to mobile: 92.5 CLP/min
  - All day, mobile to mobile: 47.2 CLP/min
- $C_{MFB}$: $240 \cdot 10^3$ CLP (Adjust)
Optimization Example
Simplified model: Poisson in-frame homogeneous
Optimization Example
Simplified model: Poisson in-frame homogeneous

Traffic to wireless devices

- **Total**
- **Bypasses**
- **Overflows**

<table>
<thead>
<tr>
<th>Number of bypasses (n)</th>
<th>Average daily Traffic [Erlang]</th>
</tr>
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<tbody>
<tr>
<td>0</td>
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<td>50</td>
</tr>
<tr>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>
Probabilites

Probabilities for the first 3 states.
System has 14 overflows and 11 bypasses

(0,0) = (bypasses, overflows)
(1,0) = (bypasses, overflows)
(2,0) = (bypasses, overflows)
Existing 14 Overflow lines

- Analytical model:
  - Optimum number of bypass: 11
  - Savings: 0.12 million USD
  - Optimum Cost: 0.348 million USD
  - Fast ROI
Transient analysis shows the difference (green is transient):

Expected Traffic (demanded and served)
Adjust values to other industries
Three state variable model
Design for Multicompany outsourcer
Find criterion for simplifications
Extend to other distributions of probability